ONE- AND TWO-DIMENSIONAL SUBWAVELENGTH SOLITONS IN SATURABLE MEDIA

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Abstract

Very narrow spatial bright solitons in (1+1)D and (2+1)D versions of cubic-quintic and full saturable models are studied, starting from the full system of the Maxwell's equations, rather than from the paraxial (NLS) approximation. For the solitons with both TE and TM polarizations, it is shown that there always exists a finite minimum width, and they cease to exist at a critical value of the propagation constant, at which their width diverges. Full similarity of the results obtained for both nonlinearities suggests that the same general conclusions apply to narrow solitons in any non-Kerr model.

1. Introduction

The standard approach to description of spatial solitons in nonlinear optical waveguides is based on the use of the nonlinear Schrödinger (NLS) equation, which replaces the Maxwell's equations (ME) in the paraxial approximation[1] (see also Refs.[2]). As is known, this approxi-

mation can be insufficient for the description of very narrow, *subwavelength* (subwavelength) solitons, with a size $\lesssim \lambda$, where λ is the carrier wavelength in vacuum[3]. Terms which are neglected when deriving the NLS equation from ME couple to the propagation constant, affecting dynamics of very narrow solitons with arbitrary polarization[3]. Particular cases of the TM (transverse-magnetic)- and TE (transverse–electric) polarized subwavelength solitons in media with purely cubic (Kerr) nonlinearity were considered in detail in Refs.[4,5]. It was found that, in the TM case, the size of both bright and dark solitons cannot be essentially smaller than $\lambda/2$. In the TE case, the size of the dark soliton is also limited from below, while the bright soliton may formally be arbitrarily narrow; however, narrow TE solitons are subject to a strong instability.

The analysis performed in those works was formal in the sense that subwavelength soliton solutions in the model with the Kerr nonlinearity imply unrealistically large values of the nonlinear correction Δn to the refractive index, $\Delta n \gtrsim 1$. As is known[1], the quadratic (in the amplitude of the electromagnetic field) correction Δn, which defines the Kerr nonlinearity, is, as a matter of fact, only the first term of the expansion in powers of the squared field. A detailed analysis shows that results of the formal consideration, based on dropping the higher-order corrections and using the ensuing truncated model in the range where the quadratic correction Δn is allowed to be large, are *ambiguous*: they strongly depend on a particular stage of the analysis at which the higher-order terms are omitted. This ambiguity is especially conspicuous in the case of the TM narrow solitons: depending on the choice of the truncation stage, one arrives at a conclusion that the width of the bright TM soliton remains limited from below, or may become arbitrarily small.

In fact, the physical problem of the existence of subwavelength solitons is unresolvable within the framework of the Kerr model, and the only possibility to obtain a definite result is to adopt a more realistic nonlinearity, with *saturation* of Δn in some form for strong fields. While an exact form of the saturation in real materials is not usually known, there are two commonly used models, viz., the saturable nonlinearity proper[6], and the cubic-quintic (CQ)[7,8] one. The former nonlinearity is realistic for alkali metal vapors[9], and the

latter nonlinearity, which combines self-focusing cubic and self-*defocusing* quintic terms, is well documented to describe nonlinear optical properties of the PTS crystal[10]. Moreover, formation of (2+1)D solitons in this material, consistent with the predictions of the CQ model, was recently observed in the experiment reported in Ref.[11]. We also note that the (2+1)D and (3+1)D versions of the CQ model have recently attracted considerable attention as they give rise to stable or almost stable multidimensional (spatial or spatiotemporal) solitons[8].

Our objective is to consider subwavelength spatial solitons of both TM and TE types *parallel* in the CQ and saturable models, in order to arrive at general conclusions concerning their width and other fundamental properties. Moreover, both (1+1)- and (2+1)-dimensional [(1+1)D and (2+1)D] cases will be considered, in the latter case the spatial soliton being a cylindrical beam. We will conclude that, in both models and for both dimensions, there always exists a finite minimum value of the soliton's width, and fundamental properties of the subwavelength solitons are fairly similar in these two models. On the basis of these results, one may infer that, in fact, qualitative results obtained in this work (first of all, the existence of the minimum width) apply to narrow spatial solitons in *any* non-Kerr model.

The rest of the paper is organized as follows. In section 2, we derive, directly from ME, general equations that describe narrow spatial TM solitons in both (1+1)D and (2+1)D geometries. In section 3, we consider a much simpler TE case. In fact, for this case soliton solutions are well known (in particular, they can be found in an exact form in the (1+1)D geometry). This allows us to rigorously prove that the relative soliton's width, divided by the carrier wavelength, cannot be smaller than a certain minimum value. In section 4, combining analytical considerations and direct numerical solutions, we find (1+1)D and (2+1)D fundamental TM solitons and arrive at the conclusion that their width is also limited from below.

2. Equations for the transverse-magnetic case

We start the analysis with the most nontrivial case of the TM polarization. In this case, irrespective of the particular nonlinearity, the *real* (physical) vectorial electric $\mathcal{E}$ and magnetic $\mathcal{H}$ fields can be taken as

$$\mathcal{H}_y = -H(x)\cos\Phi, \quad \mathcal{H}_x = \mathcal{H}_z = 0, \tag{1}$$

$$\mathcal{E}_x = E(x)\cos\Phi, \quad \mathcal{E}_z = E_z(x)\sin\Phi, \quad \mathcal{E}_y = 0 \tag{2}$$

in the (1+1)D case, and in the (2+1)D case,

$$\mathcal{H}_x = (y/r)H(r)\cos\Phi, \quad \mathcal{H}_y = -(x/r)H(r)\cos\Phi, \quad \mathcal{H}_z = 0, \tag{3}$$

$$\mathcal{E}_x = (x/r)E(r)\cos\Phi, \quad \mathcal{E}_y = (y/r)E(r)\cos\Phi, \quad \mathcal{E}_z = E_z(r)\sin\Phi; \tag{4}$$

note a phase shift $\pi/2$ between the transverse and longitudinal components in these expressions. Here, x, y and z are the transverse and propagation coordinates, $r^2 \equiv x^2 + y^2$, t is time, and the common phase of all the fields is $\Phi = \beta z - \omega t$, β and ω being, respectively, the propagation constant and frequency of the carrier wave. In the (2+1)D case, the expressions (3) and (2) correspond to the standard TM_{n1} mode in a cylindrical waveguide[12].

The simplest saturable model assumes an isotropic material characterized by the relation

$$\mathcal{D}_{\mathrm{sat}} = \varepsilon_0 \mathcal{E}\left[1 + \frac{(\varepsilon_2/\varepsilon_0)\,\mathcal{E}^2}{1 + (\varepsilon_4/\varepsilon_2)\,\mathcal{E}^2}\right] \tag{5}$$

between the electric induction and strength, with positive nonlinear permeabilities ε_2 and ε_4. Expansion and truncation of Eq. (5) yields the CQ model in the form

$$\mathcal{D}_{\mathrm{CQ}} = \mathcal{E}\left(\varepsilon_0 + \varepsilon_2\mathcal{E}^2 - \varepsilon_4\mathcal{E}^4\right). \tag{6}$$

A difference between the two models appears in the case when the truncation leading from $\mathcal{D}_{\mathrm{sat}}$ to $\mathcal{D}_{\mathrm{CQ}}$ is no longer valid. Below, we present analysis in a detailed form for the

CQ model. For the saturable one, it is quite similar, but formulas are more cumbersome. Final results will be displayed for both models together.

Following the usual rotating-wave approximation, the next step is to substitute Eqs. (1) and (2) or (3) and (4) into the relation (6) (or (5), in the case of the saturable nonlinearity), and collect all the contributions to the fundamental harmonics $\sin\Phi$ and $\cos\Phi$, neglecting higher-order harmonics. This yields direct relations between the electric-field induction and strength,

$$\mathcal{D}_{x,y} = \varepsilon_t \mathcal{E}_{x,y},\ \mathcal{D}_z = \varepsilon_l \mathcal{E}_z, \tag{7}$$

where, in the case of the CQ nonlinearity, effective nonlinear susceptibilities are

$$\varepsilon_t \equiv \varepsilon_0 + \frac{1}{4}\varepsilon_2(3E^2 + E_z^2) - \frac{1}{8}\varepsilon_4(5E^4 + 2E^2E_z^2 + E_z^4), \tag{8}$$

$$\varepsilon_l \equiv \varepsilon_0 + \frac{1}{4}\varepsilon_2(E^2 + 3E_z^2) - \frac{1}{8}\varepsilon_4(E^4 + 2E^2E_z^2 + 5E_z^4). \tag{9}$$

Insertion of the above expressions (1) and (2) or (3) for the magnetic and electric field and (4) into the Maxwell's vectorial equation for the electric field,

$$\nabla \times \mathcal{E} = -\frac{1}{c}\frac{\partial \mathcal{H}}{\partial t},$$

where c is the light velocity in vacuum, we arrive at a scalar ODE

$$E_z' + \beta E = -(\omega/c)H, \tag{10}$$

where the prime stands for d/dx or d/dr in the (1+1)D and (2+1)D cases, respectively. Further, we substitute the (1+1)D or (2+1)D expressions (1) and (2) or (3) and (4), in combination with the relations (7) and (8), (9) which define the electric induction, into the Maxwell's vectorial equation for the magnetic field,

$$\nabla \times \mathcal{H} = \frac{1}{c}\frac{\partial \mathcal{D}}{\partial t}.$$

This yields two more equations, one of which is again an ODE, while the other one is just an algebraic relation,

$$\varepsilon_t r^{1-D}(r^{D-1}H)' = (\omega/c)E_z\varepsilon_l, \tag{11}$$

$$\beta H = -(\omega/c)E, \tag{12}$$

where $D = 1, 2$ is the transverse dimension.

Eliminating the magnetic field H from Eqs. (10) and (11) by means of Eq. (12), we then obtain a system of two equations for the fields E and E_z,

$$E_z' = E(1-P), \tag{13}$$

$$r^{1-D}\left(r^{D-1}Ev_1\right)' = -E_z v_2. \tag{14}$$

Here, the variables have been rescaled as $(x,y,r) \rightarrow \beta\sqrt{\gamma}(x,y,r)$, $[E, E_z] \rightarrow \left[E, (E_z\beta/\sqrt{\gamma})\right](\omega/\beta\sqrt{\gamma})\sqrt{3\varepsilon_2}/2$, with $\gamma \equiv 1-(\beta_0/\beta)^2$, where $\beta_0 \equiv \sqrt{\varepsilon_0}\omega/c$ is the propagation constant in the linear regime, and

$$P \equiv E^2 + \frac{\gamma}{3}E_z^2 + \sigma\left(E^4 + \frac{2\gamma}{5}E^2E_z^2 + \frac{\gamma^2}{5}E_z^4\right), \tag{15}$$

$$v_1 \equiv 1 + \frac{\gamma}{1-\gamma}P, \tag{16}$$

$$v_2 \equiv 1 + \frac{\gamma}{1-\gamma}\left[\frac{1}{3}E^2 + \gamma E_z^2 + \sigma\left(\frac{1}{5}E^4 + \frac{2\gamma}{5}E^2E_z^2 + \gamma^2E_z^4\right)\right], \tag{17}$$

$$\sigma \equiv -\gamma(1-\gamma)^{-1}s, \tag{18}$$

$$s \equiv (10/9)\,\varepsilon_0\varepsilon_4\varepsilon_2^{-2}. \tag{19}$$

These equations will be used below to study TE and TM solitons in the CQ model, which will be paralleled by the same analysis for the model with the saturable nonlinearity. Before proceeding to that, it may be relevant to revisit the case of the TM soliton in the Kerr (rather than CQ) (1+1)D medium. In that case, it is necessary to bear in mind the pure cubic (Kerr) nonlinearity applies as long as the nonlinear correction to the refractive index is much smaller than the linear index. In the present notation, this condition can be shown to amount to inequalities

$$E^2(x) \ll 1 \,\text{and}\, \left(E'\right)^2 \ll 1. \tag{20}$$

Taking these into regard and performing the corresponding expansions in the above equations (13) through (17), one can eliminate the longitudinal electric field E_z and derive an eventual equation for the transverse field,

$$E'' - \gamma E + \left[E^2 + \frac{1}{3} (E')^2 \right] E = 0 \, , \tag{21}$$

where γ was defined by Eq. (**??**). In comparison with the traditional paraxial (NLS) approximation, the only new term in Eq. (21) is $(1/3)(E')^2 E$, which, as a matter of fact, is a contribution from the longitudinal component E_z in the present TM case.

Obviously, Eq. (21) can give rise to bright solitons only in the case $\gamma > 0$, which will be assumed to hold hereafter. Note that the classical broad solitons correspond to the case $\gamma \ll 1$. In that case, the broad soliton with the first correction produced by the extra term can be easily found:

$$E_{\mathrm{sol}}(x) = \sqrt{2\gamma} \left[\left(1 + \frac{\gamma}{18} \right) \mathrm{sech} \left(\sqrt{\gamma} \xi \right) - \frac{\gamma}{9} \mathrm{sech}^3 \left(\sqrt{\gamma} \xi \right) \right] . \tag{22}$$

Numerical solution of Eq. (21) shows that it does give rise to solitary-wave solutions with an arbitrary small width; however, when the soliton becomes too narrow, the solution violates the applicability conditions (20), which makes it necessary to modify the nonlinearity, i.e., to consider the model with the CQ or saturable nonlinearity, which will be done below.

3. The transverse-electric case

The above equations, derived for the CQ nonlinearity, also contain an essentially simpler case of the TE polarization, which can be obtained setting formally $\gamma = 0$ but keeping $\sigma \neq 0$. In this case, $v_1 = v_2 = 1$, $P = E^2 + \sigma E^4$, and in the (1+1)D geometry an eventual equation for $E(x)$ takes a well-known form

$$E'' = E - E^3 - \sigma E^5, \tag{23}$$

a commonly known exact soliton solution to which can be written as

$$4/E^2(x) = 1 + \sqrt{1+16\sigma/3}\cosh(2x). \tag{24}$$

It exists provided that $\sigma > -3/16$, or, with regard to Eq. (19),

$$(\beta/\beta_0)^2 < (\beta/\beta_0)^2_{\rm cr} \equiv 1 + 3/(16s). \tag{25}$$

As $(\beta/\beta_0)^2$ is approaching the value $(\beta/\beta_0)^2_{\rm cr}$, the average width of the soliton (24), defined as per Eq. (30), diverges as $|\ln[1 + 3/(16s) - (\beta/\beta_0)^2]|$, while the soliton's amplitude remains finite, $E_{\rm max} = 2$.

The main characteristic of the spatial soliton is its width. For the (1+1)D soliton (24), the FWHM[1] width Δ can be easily found,

$$\cosh\Delta = (1+16\sigma/3)^{-1/2} + 2.$$

In unnormalized units, the FWHM width is

$$W = (4\lambda/3\pi n_0)\sqrt{s}F(16\sigma/3),$$

where

$$F(y) \equiv |y|^{-1/2}\ln\left[(1+y)^{-1/2} + 2 + \sqrt{3 + 4(1+y)^{-1/2} + (1+y)^{-1}}\right]. \tag{26}$$

The function F, and hence the width of the TE soliton, attain a minimum value at $\sigma = -0.1493$. The ratio of the corresponding minimum width of the TE soliton, normalized to the carrier wavelength, is

$$W^{\rm (TE)}_{\rm min}/\lambda = 0.825\sqrt{\varepsilon_4}/\varepsilon_2\,. \tag{27}$$

In the next section, we will also use a different (*average*) definition of the width, based on Eq. (30). It is easy to find that the minimum value of the average width (30) differs from the minimum FWHM value (27) by less than 8%.

Consideration of TE solitons in the (2+1)D geometry leads to an equation

$$\frac{d^2E}{d^2r} + \frac{1}{r}\frac{dE}{dr} - \frac{1}{r^2}E = E - E^3 - \sigma E^5 \tag{28}$$

(r again being the radial variable), cf. Eq. (23). In fact, Eq. (28) is exactly the same equation which describes known solitons with an internal spin (vorticity) $s = 1$ in the CQ model in the (2+1)D geometry[8]. The origin of an effective "vorticity" term, $-E/r^2$, in Eq. (28) is the structure of the (2+1)D *ansätze* (3) and (4) for the vectorial fields.

Equation (28) has been studied in detail numerically elsewhere[8]. Although the minimum width of a family of solitons generated by this equation was not specially considered, a straightforward consequence of results presented in Refs.[8] is that the minimum width is always finite.

Thus, it is possible to rigorously prove the existence of a finite lower bound for the width of spatial TE solitons in the (1+1)D and (2+1)D models with the CQ nonlinearity. Quite similarly, the same results can be obtained for TE solitons in the model with the saturable nonlinearity (we do not display technical details here, as they do not contain anything essentially novel).

4. Transverse-magnetic narrow spatial solitons

The most interesting case is that with the TM polarization, as it does not reduce to a single-component equation. As it was mentioned above, the necessary condition for the existence of localized solutions to the corresponding equations (13) and (14), supplemented by Eqs. (15) through (19), is $\beta_0^2 < \beta^2$. Below, we will use a *relative propagation constant*, $\beta/\beta_0 \equiv (1-\gamma)^{-1/2}$, as a measure of the departure from the paraxial approximation: in the limit of a very broad soliton, one has $\beta/\beta_0 \to 1$, while in the opposite limit of an infinitely narrow soliton (if any), $\beta/\beta_0 \to \infty$.

A straightforward analysis of Eqs. (13) and (14) makes it possible to prove that, in the (1+1)D case for both CQ and saturable nonlinearities, localized solutions with no zeros of $E(x)$ and $E_z(x)$ at $x \neq 0$ do not exist if $E(0) = 0$. Further analysis, details of which are not displayed here, shows that, in the (1+1)D geometry, a localized fundamental-soliton solution has an even monotonically decreasing transverse component $E(|x|)$, while the longitudinal

one $E_z(x)$ is odd, with $E_z(0) = 0$ and a single extremum at finite $|x|$, see Fig. 1a (in this and next figures, the results are simultaneously shown for the CQ and saturable models). In the (2+1)D case, fundamental solitons feature a different structure: they have $E(0) = 0$, and both $E(r)$ and $E_z(r)$ may have zeros at finite r, see Fig. 1b.

Proceeding to the width of the TM soliton, we use the electromagnetic energy density,

$$\mathcal{Q} = \mathcal{E}_x\mathcal{D}_x + \mathcal{E}_y\mathcal{D}_y + \mathcal{E}_z\mathcal{D}_z + \mathcal{H}_x\mathcal{H}_x + \mathcal{H}_y\mathcal{H}_y \, , \tag{29}$$

to define the soliton's average half-width as

$$W = \int_0^\infty \mathcal{Q}r^D dr / \int_0^\infty \mathcal{Q}r^{D-1} dr. \tag{30}$$

The FWHM definition of W, which was used in the previous section for the TE soliton, is ambiguous for TM solitons, as they have three different components. The results for the average width, which were obtained from numerical solutions of Eqs. (13) and (14), are summarized in Fig. 2, showing W vs. the relative propagation constant at different values of the material constant s defined by Eq. (19). Basic features of the dependence are the same as those following from the above analytical expressions for the TE soliton in the (1+1)D case: there is a *finite minimum* of the soliton's width, and the solitons do not exist beyond a critical value of β/β_0, at which the soliton's width diverges while its amplitude remains finite, cf. Eq. (25). Naturally, the minimum value of the width depends on the dimension of space, type of the nonlinearity, and, for a fixed nonlinearity, on values of material constants. As well as in the case of the TE solitons, the minimum width tends to zero and the maximum value of β/β_0 diverges as $\varepsilon_4 \to 0$.

Note that the soliton shapes shown in Fig. 1 pertain to the fixed value of the relative propagation constant $\beta/\beta_0 = 1.05$, which is rather close to the paraxial limit. Comparison with other numerical results demonstrates that, for both CQ and saturable models, the shapes of the (1+1)D and (2+1)D TM solitons remain fairly similar to those shown in Fig. 1 with the increase of β/β_0 up to the point where the minimum width is attained. With the subsequent increase of β/β_0 up to the value at which the solitons cease to exist, the shape changes much more.

The above considerations did not tackle the stability problem. While a consistent stability analysis requires very tedious simulations of the Maxwell's equations with the CQ or saturable nonlinearity, which is beyond the scope of this work, we tried to test the stability by means of the simple Vakhitov-Kolokolov (VK) criterion[13] (although its applicability to the present model is not obvious): the propagation constant must have a positive slope as a function of the beam's power (norm of the solution),

$$N = 2^{2-D} (2\pi)^{D-1} \int_0^\infty \left(\mathcal{E}_x^2 + \mathcal{E}_y^2 + \mathcal{E}_z^2 \right) r^{D-1} dr$$

(cf. Eq. (30)), which is a dynamical invariant of the model. Our numerical calculations have shown that all the TM solitons, in both the (1+1)D and (2+1)D geometries, satisfy the VK criterion.

5. Conclusion

In this work, we have studied very narrow (1+1)D and (2+1)D spatial TE and TM solitons supported by the quintic-cubic and saturable nonlinearities, starting the full Maxwell's equations. For TE solitons, we have obtained a single equation, which is equivalent to the stationary version of the nonlinear Schrödinger equation with the corresponding nonlinearity, while for TM solitons we end up with a system of equations for the transverse and longitudinal components of the electric field, rather than with a single one. It is also noteworthy that TE solitons in the (2+1)D case are described by the same equation as vortex solitons with spin $s = 1$ in the (2+1)D nonlinear Schrödinger equation.

Detailed analysis was displayed for the cubic-quintic model, while final results were given for the saturable model as well. Full qualitative similarity of the results obtained for both models (although exact scales may be quite different, see caption to Fig. 2) makes it very plausible that the situation is basically the same for *any* physically relevant non-Kerr medium. The most important conclusions are that, in all the cases, there is a finite minimum of the soliton's width, and the solitons cease to exist at a critical value of the propagation constant, at which their width diverges.

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Figure captions

Fig. 1. Typical examples of the fundamental spatial TM soliton in the (1+1)D (a) and (2+1)D (b) geometries. The relative propagation constant is $\beta/\beta_0 = 1.05$, and the material constant (19) is $s = 1$. In this and next figures, the solid and dotted curves pertain, respectively, to the cubic-quintic and saturable models.

Fig. 2. The width of the (1+1)D (a) and (2+1)D (b) fundamental spatial TM solitons vs. the relative propagation constant β/β_0 at various fixed values of the material constant s. For the saturable model, the deviation of the relative propagation constant from 1 is five times that shown on the horizontal axis for the cubic-quintic model.